HW SOL 3.4

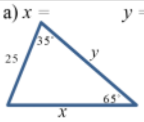
May 13, 2018 10:38 PM

Name:______________________ Date:______________________

Math 10/11 Honors Section 3.4 Sine Law and Cosine Law

1. Given each triangle, find the value of any missing side or angle "x" and "y"

a) $x =$ $y =$



① $\frac{\sin 35°}{x} = \frac{\sin 65}{25}$

$\frac{25 \cdot \sin 35}{\sin 65} = x$

$15.82 = x$

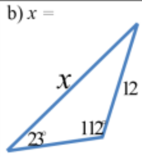
② $180 - 35 - 65 = 80$

③ $\frac{\sin 80}{y} = \frac{\sin 65}{25}$

$\frac{25 \cdot \sin 80}{\sin 65} = y$

$27.165 = y$

b) $x =$

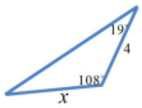


$\frac{\sin 23°}{12} = \frac{\sin 112}{x}$

$x = \frac{12 \times \sin 112}{\sin 23°}$

$x = 28.47$

c) $x =$



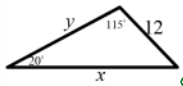
① $180 - 108 - 19 = 53°$

② $\frac{4}{\sin 53°} = \frac{x}{\sin 19}$.

$\frac{4 \sin 19}{\sin 53} = x$

$1.63 = x$

d) $x =$ $y =$



① $\frac{12}{\sin 20°} = \frac{x}{\sin 115}$.

$\frac{12 \times \sin 115}{\sin 20} = x$

$31.798 = x$

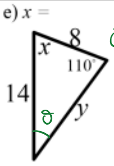
② $180 - 115 - 20 = 45°$

③ $\frac{y}{\sin 45} = \frac{12}{\sin 20}$

$y = \frac{12 \times \sin 45}{\sin 20°}$

$y = 24.809$

e) $x =$ $y =$



① $\frac{\sin 110°}{14} = \frac{\sin\theta}{8}$

$\frac{8 \times \sin 110°}{14} = \sin\theta$

$0.536967 = \sin\theta$

$32.48° = \theta$

② $x = 180 - 110 - 32.48$

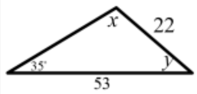
$x = 37.52°$

③ $\frac{y}{\sin 37.52} = \frac{14}{\sin 110}$

$y = \frac{14 \times \sin 37.52}{\sin 110}$

$y = 9.074$ //

f) $x =$ $y =$



This side is too big for this triangle to exist!

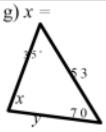
① $\frac{\sin x}{53} = \frac{\sin 35°}{22}$

$\sin x = \frac{53 \times \sin 35}{22}$

$x = \sin^{-1}(1.38179)$

Not possible (No Answer).

g) $x =$ $y =$



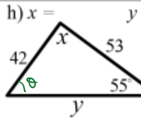
① $x = 180 - 70 - 35 = 75°$

② $\frac{y}{\sin 35} = \frac{53}{\sin 75}$

$y = \frac{53 \times \sin 35}{\sin 75}$

$y = 31.47$ //

h) $x =$ $y =$



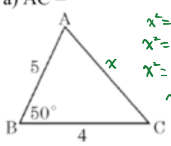
$\frac{\sin 55}{42} = \frac{\sin\theta}{53}$

$\frac{53 \times \sin 55}{42} = \sin\theta$

$1.03369 = \sin\theta$

Not possible (No Answer)

a) AC =



$x^2 = 5^2 + 4^2 - 2(5)(4)\cos 50°$

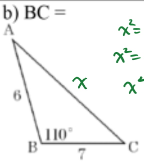
$x^2 = 25 + 16 - 40 \cos 50°$

$x^2 = 41 - 40(0.642787)$

$x^2 = 15.2884951$

$x = 3.91$ //

b) BC =



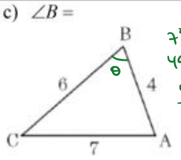
$x^2 = 6^2 + 7^2 - 2(6)(7)\cos 110°$

$x^2 = 36 + 49 - 84(-0.342020)$

$x^2 = 113.72969$

$x = 10.66$ //

c) $\angle B =$



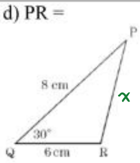
$7^2 = 6^2 + 4^2 - 2(6)(4)\cos\theta$

$49 = 36 + 16 - 48\cos\theta$

$\frac{49-52}{-48} = \cos\theta$

$86.42^\circ = \theta$

d) PR =



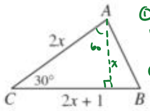
$x^2 = 8^2 + 6^2 - 2(8)(6)\cos 30^\circ$

$x^2 = 64 + 36 - 96\left(\frac{\sqrt{3}}{2}\right)$

$x^2 = 100 - 48\sqrt{3}$

$x = 4.106$ //

2. In the diagram, $AC = 2x$, $BC = 2x+1$ and $\angle ACB = 30^\circ$. If the area of $\triangle ABC$ is 18, what is the value of "x"?



① HEIGHT = x
b/c 30, 60, 90 Δ.

② $A = \frac{\text{BASE} \times \text{HEIGHT}}{2}$

$\frac{(x)(2x+1)}{2} = 18$

$2x^2 + x = 36$

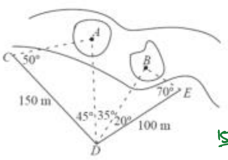
$2x^2 + x - 36 = 0$

2 — −4
1 — 9

$(2x+9)(x-4) = 0$

$x \neq -\frac{9}{2}$ $\boxed{x=4}$

3. In the diagram, points A and B are located on islands in a river full of rabid aquatic goats. Determine the distance from A to B, to the nearest meter.



① IN Δ CAD, FIND AD.

$\angle A = 180 - 50 - 45$

$\angle A = 85$

$\frac{150}{\sin 85} = \frac{AD}{\sin 50}$

$\frac{150 \times \sin 50}{\sin 85} = AD$

$115.345913 = AD$

② IN Δ BED, FIND BD

$\angle B = 180 - 70 - 20$

$\angle B = 90$

$\sin 70 = \frac{BD}{100}$

$100 \times \sin 70 = BD$

$93.96926208 = BD$

③ USE COSINE LAW

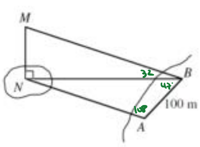
$AB^2 = AD^2 + BD^2 - 2(AD)BD\cos 35$

$AB^2 = 22{,}134.82765 - 17{,}757.48$

$AB^2 = 4377.3476$

$AB = 66.16$ m //

4. In determining the height, MN, of a tower on an island, two points A and B, 100 meters apart, are chosen on the same horizontal plane as "N". If $\angle NAB = 108^\circ$, $\angle ABN = 47^\circ$, and $\angle MBN = 32^\circ$, determine the height of the tower to the nearest meter.



① FIND BN.

$\angle BNA = 180 - 108 - 47$
$= 25^\circ$

$\frac{BN}{\sin 108} = \frac{100}{\sin 25}$

$BN = \frac{100 \times \sin 108}{\sin 25}$

$BN = 225.0391$

② FIND $\overline{MN}$.

$\tan 32^\circ = \frac{MN}{BN}$

$225.039 \times \tan 32 = MN$

$140.62 = MN$

5. In triangle ABC, $\angle ABC = 45°$. Point "D" is on $\overline{BC}$ so that $2 \cdot BD = CD$ and $\angle DAB = 15°$. Find $\angle ACB$.

a) 54° b) 60° c) 72° $\boxed{\text{d) 75°}}$ e) 90°

① BDA = 120°

② In a triangle, if none of the sides have a length, pick a side & give it a length. ie: make AD = 1.

③ Now solve for x.

$\frac{x}{\sin 15} = \frac{1}{\sin 45}$

$x = \frac{\sin 15}{\sin 45}$

④ $2x = \frac{2 \times \sin 15}{\sin 45} = 0.732$

⑤ Find AC.

$AC^2 = 1^2 + 0.732^2 - 2(1)(0.732)\cos 60$

$= 1.535898385 - 0.732$

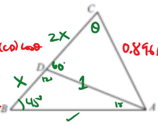
$AC = 0.8966038$

⑥ $1^2 = AC^2 + CD^2 - 2(AC)(CD)\cos\theta$

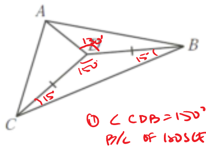
$\frac{1 - AC^2 - CD^2}{-2(AC)(CD)} = \cos\theta$

$0.2588108 = \cos\theta$

$\boxed{75° = \theta}$



6. In the diagram, $DC = DB, \angle DCB = 15°$, and $\angle ADB = 130°$. What is the measure of $\angle ADC$?

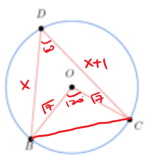


① ∠CDB = 150° b/c of isosceles triangle.

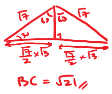
② 360 − 130 − 150
= 80°

Angles at a point

7. In the diagram, the circle has radius $\sqrt{7}$ and centre O. Points D, B, and C are on the circle. If $\angle BOC = 120°$ and $DC = DB + 1$, determine the length of DB.



① Find BC.



BC = √21

② Cosine law.

$BC^2 = x^2 + (x+1)^2 - 2(x)(x+1)\cos 60°$

$21 = x^2 + x^2 + 2x + 1 - x(x+1)$

$21 = \cancel{x^2} + x^2 + \cancel{2x} + 1 - \cancel{x^2} - \cancel{x}$

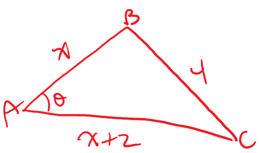
$21 = x^2 + x + 1$

$0 = x^2 + x - 20$

$0 = (x+5)(x-4)$

$x \neq -5, \quad \underline{\underline{x = 4}}$

8. In $\triangle ABC, BC = 4, AB = x, AC = x + 2,$ and $\cos(\angle BAC) = \frac{x+8}{2x+4}$. Determine all possible values of "x".



$4^2 = x^2 + (x+2)^2 - 2(x)(x+2)\cos\angle A$

$16 = x^2 + x^2 + 4x + 4 - \cancel{2}x\cancel{(x+2)}\frac{(x+8)}{\cancel{2}\cancel{(x+2)}}$

$16 = 2x^2 + 4x + 4 - x^2 - 8x$

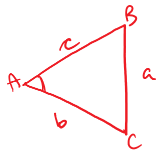
$16 = x^2 - 4x + 4$

$0 = x^2 - 4x - 12$

$0 = (x-6)(x+2)$

$x = 6 \quad x \neq -2$

9. In $\triangle ABC$, $BC=a$, $AC=b$, $AB=c$, and $a<\frac{1}{2}(b+c)$. Prove that $\angle BAC<\frac{1}{2}(\angle ABC+\angle ACB)$



① Given: $a<\frac{1}{2}(b+c)$

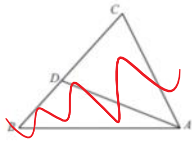
$2a<b+c$

② Prove

$\angle A<\frac{1}{2}(\angle B+\angle C)$

~~10. In triangle ABC, $\angle ABC=45°$. Point "D" is on $\overline{BC}$ so that $2 \cdot BD=CD$ and $\angle DAB=15°$. Find $\angle ACB$~~

~~a) 54° b) 60° c) 72° d) 75° e) 90°~~



10. Challenge: In the diagram, $2\angle BAC=3\angle ABC$ and "K" lies on BC such that $\angle KAC=2\angle KAB$. Suppose that $BC=a$, $\cancel{AB}=b$, $AB=c$, $AK=d$, and $BK=x$

a) Prove that $d=\frac{bc}{a}$ and $x=\frac{a^2-b^2}{a}$ (Euclid)

$\frac{a}{b}=\frac{b}{a-x}$

$a^2-ax=b^2$

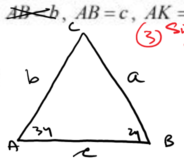
$a^2-b^2=ax$

$\frac{a^2-b^2}{a}=x.$

$\frac{a}{c}=\frac{b}{d}$

$d\times a=bc$

$\boxed{d=\frac{bc}{a}}$

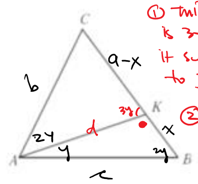


③ Similar Triangles

$a\sim b$

$b\sim a-x$

$c\sim d.$



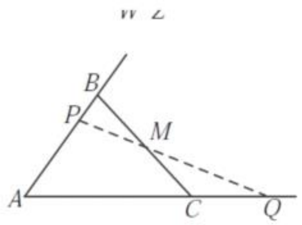
① This angle is 3y b/c it supplementary to 3rd angle

② Use similar triangles

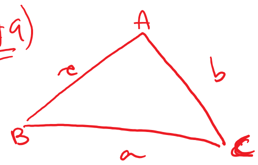
b) Prove that $(a^2-b^2)(a^2-b^2+ac)=b^2c^2$ (Super Challenge: Requires Cosine Law)

Euclid 2007 #10b Super Challenging! Requires Cosine Law

(b) In the diagram, $AB=10$, $BC=14$, $AC=16$, and M is the midpoint of BC. Various lines can be drawn through M, cutting AB (possibly extended) at P and AC (possibly extended) at Q. Determine, with proof, the minimum possible perimeter of $\triangle APQ$.



#9)



$\boxed{a^2=b^2+c^2-2bc(\cos\angle A)}$

① Given

$2a<b+c$

② Goal Prove

$\angle A<\frac{1}{2}(B+C)$

③ $\angle A+\angle B+\angle C=180°$

$\angle A=180°-\angle B-\angle C$

$\boxed{\angle B+\angle C=180°-\angle A}$

$\angle A<\frac{1}{2}(180-\angle A)$

$2\angle A<180-\angle A$

$3\angle A<180°$

$\boxed{\angle A<60°}$

Goal: To prove $\angle A$ is less than 60°

④ $2a<b+c$

$4a^2<b^2+2bc+c^2$

$4(b^2+c^2-2bc(\cos\angle A))<b^2+c^2+2bc$

$4b^2+4c^2-8bc(\cos\angle A)<b^2+c^2+2bc+3(b-c)^2$

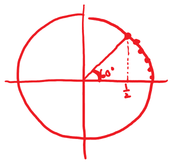
$4b^2+4c^2-8bc(\cos\angle A) < b^2+c^2+2bc+3(b^2-2bc+c^2)$

$\cancel{4b^2}+\cancel{4c^2}-8bc(\cos A) < \cancel{4b^2}+\cancel{4c^2}-4bc$

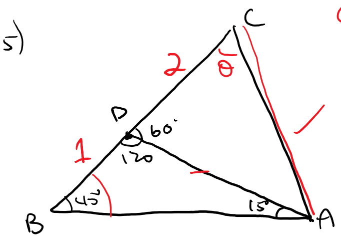
$\frac{-8bc(\cos A)}{-8bc} < \frac{-4bc}{-8bc}$

$\cos A > \frac{1}{2}$

$\therefore A < 60°$



#5)



① Find BA.

$\frac{1}{\sin 15°} = \frac{BA}{\sin 120°}$

$\boxed{\frac{\sin 120}{\sin 15} = BA}$

② Cosine Law

$CA^2 = 3^2 + BA^2 - 2(3)(BA)\cos 45°$

$CA^2 = 9 +$

$CA^2 = 6.01\ldots$

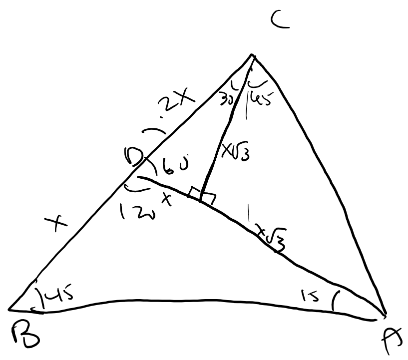
$CA = 2.45\ldots$

③ $\frac{\sin 45°}{CA} = \frac{\sin\theta}{BA}$

$\frac{BA \times \sin 45°}{CA} = \sin\theta$

$\_\_\_\_\_\_ = \sin\theta$

$75° = \theta$ //



① Find $\overline{DA}$

$\frac{DA}{\sin 45°} = \frac{x}{\sin 15°}$

$DA = x\left(\frac{\sin 45}{\sin 15}\right)$

$DA = x(1+\sqrt{3})$